FRACTALS – return to origin

Fraktale – powrót do źródeł

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ABSTRACT: Fractals have become fashionable. Therefore, in recent years there have been many articles in which the authors support something that they call a fractal account, including, for example, the sum of fractal dimensions. This paper is a recapitulation of what a fractal account is in the Earth sciences, what are its uses and boundaries. The definition of fractals is: it has a non trite structure in all scales, it is very hard to describe fractal structure in the Euclidean geometry, it is self-similar (directly or statistical), its Hausdorf dimension is greater than its topological dimension, it is described by recurrent formula, its dimension is not an integral number. In the face of such a wide and imprecise formula, various fields of science have introduced their definitions of fractal. It only has to meet most of the conditions included in the definition. In the analysis of geological objects in Earth sciences and in oil and gas industry, fractals are defined by the recurrence formula with its range of applicability, fractal dimension share a part of the space occupied by the fractal object, so the highest value of fractal dimension is equal to 3. Fundamental work in which the name of fractals for self-similar objects were introduced was *The Fractal Geometry of Nature* by Mandelbrot (1977). In the Earth sciences, statistical fractals (pseudofractals) are used. The straight line in the log-log plot is the indicator of fractal structure. In other words, the fractal structures are associated with the power patterns obtained during the analysis of geological objects. Generally, in the analysis of geological objects the Menger sponge and box methods of fractal dimension calculations are used. Fractals provide a unique opportunity to characterize complicated objects with the use of a single number, nevertheless, in order for the obtained results to be applicable and comparable with the results of other analyzes, both the model of the analyzed object and the method of calculation of the fractal dimension should be given, as well as the scope of applicability of this dimension.

Key words: fractal dimension, geological objects, Menger sponge, box dimension.

STRESZCZENIE: Fraktale stały się modne. W związku z tym w ostatnich latach obserwuje się wiele artykułów, w których autorzy wspierają się czymś, co nazywają rachunkiem fraktalowym, łącznie np. z sumowaniem wymiarów fraktalowych. Niniejszy artykuł stanowi rekapitulację tego, czym jest rachunek fraktalowy w naukach o Ziemi, jakie są jego zastosowania i granice. Co jest, a co nie jest fraktalem. Zasadniczo na definicję wymiaru fraktalnego składają się następujące warunki: nie jest prostą i taką samą strukturą we wszystkich skalach, bardzo trudno opisać go w geometrii euklidesowej, jest strukturą samopodobną (wprost lub statystycznie), jego wymiar Hausdorffa jest większy od jego wymiaru topologicznego, jest opisany formułą rekurencyjną oraz jego wymiar nie jest liczbą całkowitą. Wobec tak szerokiej i nieprecyzyjnej formuły różne dziedziny nauki wprowadziły swoje definicje fraktala. Ma on jedynie spełniać większość warunków zapisanych w definicji. W naukach o Ziemi fraktale definiowane są przez wzory rekurencyjne z analizą obszaru stosowalności. W analizie obiektów geologicznych wymiar fraktalny wskazuje na część przestrzeni zajmowaną przez dany obiekt, w związku z czym jego wartość nie może przekraczać 3. Mandelbrot w swojej fundamentalnej pracy *The Fractal Geometry of Nature* (1977) wprowadził nazwę fraktala jako obiektu samopodobnego. W naukach o Ziemi stosowane są fraktale statystyczne, zwane również pseudofraktalami. Wskaźnikiem struktury fraktalnej jest linia prosta na wykresie typu log-log. Inaczej mówiąc, fraktalne struktury są związane ze wzorami potęgowymi uzyskanymi podczas analizy obiektów geologicznych. Zasadniczo w analizie obiektów geologicznych stosujemy model gąbki Mengera oraz wymiar pudełkowy dla obiektów dwuwymiarowych. Fraktale dają unikalną możliwość scharakteryzowania skomplikowanych struktur za pomocą jednej liczby. Tym niemniej, aby otrzymane wyniki były stosowalne i porównywalne z wynikami innych analiz, należy zarówno podać model analizowanego obiektu i sposób wyliczenia wymiaru fraktalnego, jak też określić zakres stosowalności tego wymiaru. Tylko wtedy wymiar fraktalny będzie miał sens fizyczny.

Słowa kluczowe: wymiar fraktalny, obiekty geologiczne, gąbka Mengera, wymiar pudełkowy.

Introduction

Fractal approach has become a standard in the Earth sciences. It is applied in the analysis of faults, pore space, fractures. Practically only statistical fractals (pseudofractals) are used in the analysis of geological objects (Mandelbrot, 1977; Such, 2002; Such et al., 2016). This fact does not remove definition restriction but adds a new one. It is the range of applicability. This paper reminds us of the base definitions and methods of calculating fractal dimension, presents workflows and shows that it is impossible to obtain a fractal dimension greater than 3 in the analysis of geological objects (Sokołowska et al., 1989; Cicha-Szot et al., 2015).

Fractals and fractal dimension

Because of a great variety of fractal applications, mathematicians haven't formulated a strict definition of fractals, but only proposed to describe fractals as a set which (Mandelbrot, 1977; Kudrewicz, 1993):
- Has non trite structure in all scales;
- It is very hard to describe fractal structure in the Euclidean geometry;
- It is self-similar (strictly of statistical);
- Its Hausdorf dimension is greater than its topological dimension;
- Is described by a recurrent formula;
- Its dimension is not an integral number.

An investigated object could be classified as fractals when it satisfies only most of the above conditions. And now, it is possible to define what fractals mean in the analysis of geological objects.

Fractals in the Earth sciences

Mandelbrott (1977) in his fundamental work *The Fractal Geometry of Nature,* named self-similar objects fractals, and showed that many processes and objects in nature are described with their use.

In the Earth science (Garrison et al., 1991; Angulo et al., 1992; Acuna et al., 1995), in the analyses of geological objects, fractals are defined by the recurrence formula with a range of applicability, fractal dimension share a part of space occupied by fractal object, so the highest value of the fractal dimension is equal to 3.

Deterministic fractals

Deterministic fractal is a structure defined by formula (1):

$$A = \frac{C}{S^D} \tag{1}$$

where:
A – number of fragments,
C – proportionality constants,
S – specific dimension (1/S – reduction coefficient),
D – fractal dimension.
Finally:

$$D = \frac{\log(A)}{\log\left(\frac{1}{S}\right)} \tag{2}$$

Statistical fractals

The definition of statistical fractal (pseudofractal) is as follows: for such types of objects, fractal structures can be recognized in relation to the characteristic dimension – size distribution. In other words, if the size distribution shows the fractal structure, number N is the value of distribution function greater than $S,$ and ought to fit the following equation:

$$A = \frac{1}{S^D} \tag{3}$$

There are several additional restrictions in statistical fractals (pseudofractal) introduction in comparison with pure abstract fractals. Statistical fractals are self-similar only in the appropriate range of size. This range could be also a function of the investigation method (e.g. when mercury porosimetry is applied in pore space investigations capillary forces restrict range of self-similarity). These restriction must be discussed for each measured object.

Generally the straight line in the log-log plot is an indicator of fractal structure. In other words statistical fractals are connected with power law in the analyses of geological objects.

Generalized Menger's sponge

Generalized Menger's sponge and box dimension are used in fractal approach devoted to geological objects. Menger's sponge is used in pore space analysis.
For it:

$$V \approx P^{(3-D)} \tag{4}$$

where:
V – pore space volume,
P – capillary pressure,
D – fractal dimension.

This function is obtained from porosimetric measurements. After logarithmization, in a log-log plot a straight line chart is obtained:

$$\log(V) = (3 - D) \cdot \log(P) \qquad (5)$$

The pore size distribution cumulative curve is shown in Figure 1A. Figure 1B presents its log-log version. A part of this curve (in red) manifests a fractal.
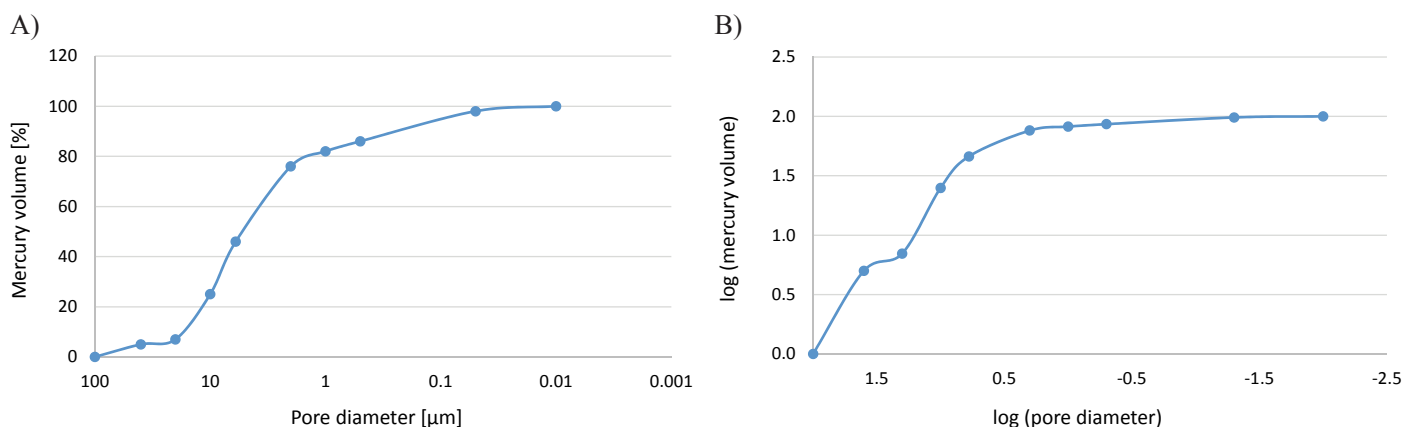
The straight line of this log-log plot gives the fractal criterion. Fractal dimension is calculated from the direction coefficient of the straight line. Detailed considerations of Menger's sponge application are presented in the works (Katz and Thompson, 1984; Garrison et al., 1991; Kudrewicz, 1993; Patzek, 1999; Such, 2002).

The sketch of Menger sponge application in fractal approach is presented in Figure 2.

Fig. 1. Typical pore size distribution cumulative curve (A) and log-log plot of this curve (B)
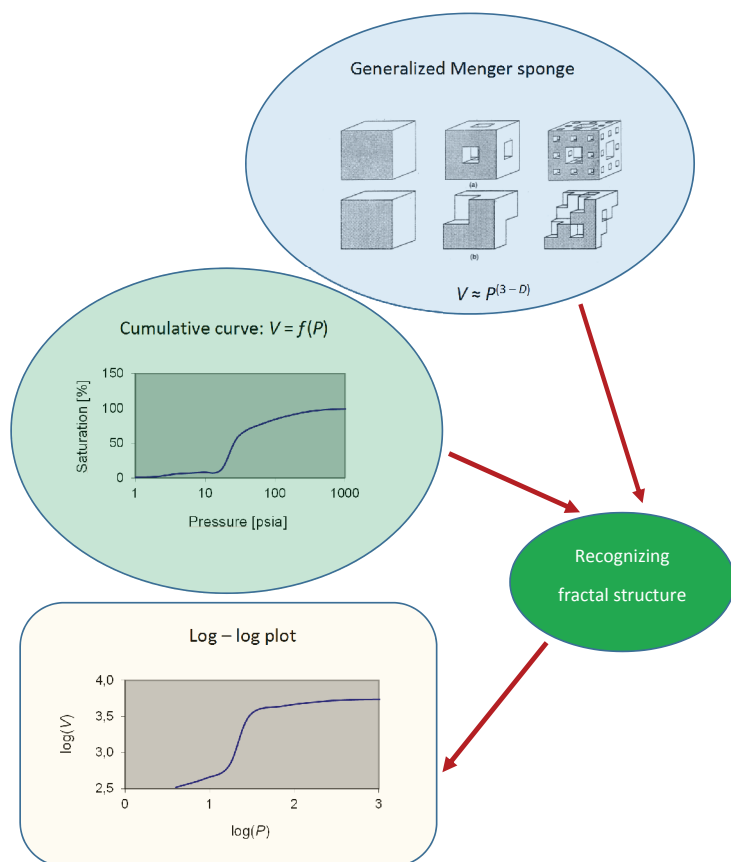**Rys. 1.** Typowa krzywa rozkładu średnic porów (A) i wykres log-log dla tej krzywej (B)

Fig. 2. Sketch of introduction of Menger sponge fractal dimension
Rys. 2. Schemat obliczenia wymiaru fraktalnego z zastosowaniem modelu gąbki Mengera

Box fractal dimension

The power law criterion is also used in the so called box dimension. It depends on the introduction of a sequence of net boxes whose size is two times smaller, for n equal to $k + 1$ when compared with n equal to k.
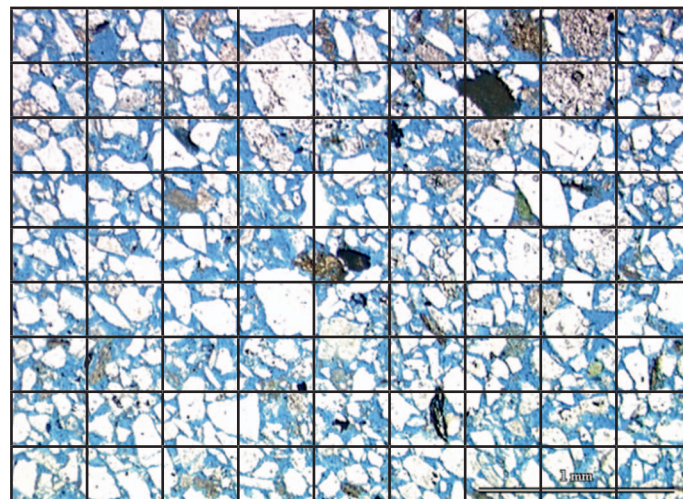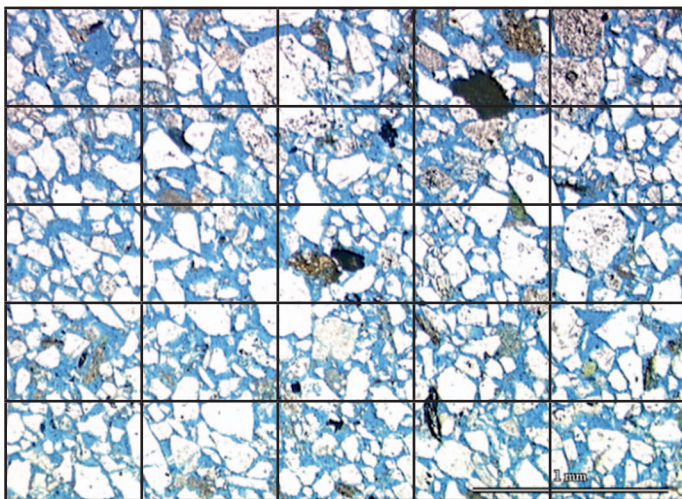
We can now prepare a log-log plot: on x axis is a number of boxes, on Y one number of boxes which have contact with the analyzed object. It is illustrated in Figure 3. We can describe it as a formula (Such, 2002):

$$\frac{\log N(2^{-(k+1)}) - \log N(2^{-k})}{\log 2^{(k+1)} - \log 2^{k}} = \log_2 \frac{N(2^{-(k+1)})}{N(2^{-k})} \qquad (6)$$

Where k is a box generation number.

Using the box counting method, fractal dimension is again the slope of the line when we plot the value of **log(N)** on the Y-axis against the value of **log(r)** on the X-axis. N is the number of boxes that cover the pattern, and r is the magnification, or the inverse of the box size. The same equation is used to define the fractal dimension, **D**. If during transition from k to $k + 1$ generation boxes number N increases in factor equal 2^D, it means that the fractal number is D. Finally box fractal dimension is equal:
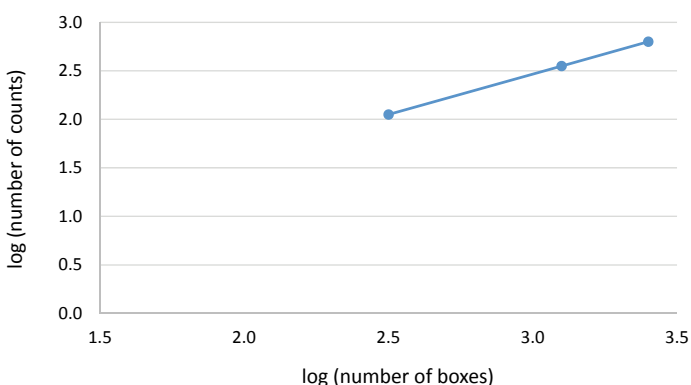
$$D = \frac{\log(N)}{\log(r)} \qquad (7)$$

Fig. 3. Microscopic image of a thin section of porous rock. *K* and *k* + 1 generation of boxes marked (G. Leśniak – unpublished)

Rys. 3. Mikroskopowy obraz płytki cienkiej dla porowatej skały. Na obrazie zaznaczono generację „pudełek" *k* i *k* + 1 (G. Leśniak – materiał niepublikowany)

The slope of the log-log line shows the box fractal dimension. Such a plot is show in Figure 4.

Fig. 4. Calculation of box fractal dimension (number of counts is equal to *N*)

Rys. 4. Sposób obliczania fraktalnego wymiary pudełkowego (liczba zaliczeń na osi *Y* jest równa *N*)

Fractals are not a curiosity

Fractal dimension gives a unique opportunity to characterize very complicated structures (such as pore space) with the use of a single number. It is the basic tool in rock typing, in correlation analyses (for example fractal dimension – residual water saturation), analyses of pore space damages and fractal dimension. Also the single value of fractal dimension divides rocks from analyzed formation into permeable and impermeable. Various types of porous media can be analyzed like soils (Sokołowska, 1989; Sokołowska et al., 1989) or nanopore space in which adsorption processes dominates (Cicha-Szot et al., 2015; Such et al., 2016). Box fractal dimension give information about faults and fracture shapes, about the number of fracture systems (and their parameters) in rock beds

and about pore space heterogeneity (Cicha-Szot et al., 2015, Such et al., 2016).

There are two necessary condition for fractal approach applications. The results must be comparable with results obtained for other geological objects and the results must have physical meaning. Satisfying these conditions require applying methods of fractal dimension calculation described in this paper, defining the numerical model of the analyzed object as well as discussing ranges of applicability of pseudofractals

Discussion

Fractals are very broad class of objects. Now the type of fractal definition depends frequently on the field of knowledge. So we have introduced a fractal approach in certain field of knowledge (here in geoscience), we must defined what a fractal is. It means we must strictly defined the self-similarity, methods of recognizing such structures and their physical meaning (Sokołowska et al., 1989; Cicha-Szot et al., 2015; Such et al., 2016).

So finally in the Earth science during fractal analyses of geological objects (Sokołowska, 1989; Peitgen et al., 1995; Turcotte, 1997; Such, 1998; Such, 2002) we can assume:

1. Fractal dimension is a measure of what part of space is taken up by fractal.
2. Fractal dimension must be lower than or equal to 3.
3. Fractal structures are recognized by power low equations.
4. It is necessary to define ranges of self-similarity as well as to describe methods of calculating fractal dimension.
5. If it is necessary to compare various objects the model of the analyzed structure must be shown (for example net model of pore space).

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