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Tadeusz Szpunar, Paweł Budak

Instytut Nafty i Gazu – Państwowy Instytut Badawczy

Mariusz Dziadkiewicz

Polskie Górnictwo Naftowe i Gazownictwo S.A.

A simple technique for approximate evaluation of permeability and skin of a dry gas zone with low to moderate permeability using wellhead pressure data

This paper presents a technique for approximate evaluation of permeability and skin of a dry gas zone with low to moderate permeability. The herein given technique may be used for analyzing the wellhead buildup pressure data of a gas well. The results obtained using the presented technique are approximate, because of some simplifying assumptions of the mathematical model and use of the classical method of downhole pressure calculation in static gas column, which is known to be of rather moderate accuracy. In a computer program (not shown) we used the trial and error method to improve the accuracy of down hole pressure calculation and for the evaluation of the average value of gas deviation factor. In the case of thick, highly permeable gas zones, the duration of wellhead pressure build up time may be too short to obtain reliable results. The procedure of execution of the presented technique is very similar to that of the well known "slug test" method, which is used for the evaluation of permeability and skin of reservoirs which do not flow to the surface, or for analyzing the drill stem test flow period data. Contrary to "Horner type" analysis of pressure build up data, neither the flow rate of gas nor the flow duration, need not be known. The procedure of permeability calculation is shown using five examples of gas wells from the domestic oil industry. To facilitate calculations all equations were converted to the engineering system of units.

Key words: gas well, wellhead buildup pressure, bottom hole pressure, diffusivity equation, iteration procedure.

Nowa metoda interpretacji danych odbudowy ciśnienia głowicowego w odwiertach gazowych udostępniających złoże o niskiej lub umiarkowanej przepuszczalności

W artykule zaproponowano sposób obliczania przepuszczalności i skin efektu odwiertu gazowego o niskiej i umiarkowanej przepuszczalności warstwy gazonośnej. Niniejszy sposób można zastosować do analizy krzywej odbudowy ciśnienia głowicowego po krótkotrwałej eksploatacji gazu z odwiertu. W przeciwieństwie do interpretacji danych metodą Hornera nie jest potrzebna znajomość wydatku gazu oraz czasu, przez jaki wydatek ten był utrzymywany. Podano model matematyczny leżący u podstaw proponowanej metody oraz pięć przykładów obliczeń dla odwiertów z krajowego przemysłu naftowego. Należy podkreślić, że do obliczeń przepuszczalności ani wydatek gazu podczas wypływu z odwiertu, ani sumaryczna jego objętość i czas trwania wypływu nie muszą być znane, natomiast konieczna jest znajomość parametrów i składu gazu oraz pojemności odwiertu z uwagi na użycie metod bilansu masowego zamiast zasady superpozycji rozwiązań przyjętej w metodzie Hornera. W przypadku grubych, wysoce przepuszczalnych stref gazowych czas narastania ciśnienia w odwiercie może być zbyt krótki, aby uzyskać wiarygodne wyniki. W programie komputerowym (niezaprezentowany) wykorzystano metodę „prób i błędów", aby poprawić dokładność obliczeń ciśnienia dennego w odwiercie oraz oszacowania średniej wartości współczynnika ściśliwości gazu. Procedura interpretacji danych za pomocą prezentowanego sposobu jest bardzo podobna do powszechnie znanej metody slug test, która jest używana do oceny przepuszczalności i skin efektu dla złóż cieczy, z których nie ma wypływu na powierzchnię, lub do analizy danych uzyskiwanych podczas opróbowań otworów. Równania przyjęte do obliczeń zostały przeliczone z systemu jednostek SI na system jednostek przyjmowany w przemyśle naftowym.

Słowa kluczowe: odwiert gazowy, odbudowa ciśnienia głowicowego, ciśnienie denne, równanie dyfuzji, procedura iteracyjna.

Consider a dry gas well in which the wellhead pressure has stabilized at p_{who} level and the stabilized bottom hole pressure was p_{bho}. Imagine that opening the wellhead valve for a while and producing the gas has partially "unloaded" the well and – after the well is closed – the wellhead pressure dropped to $p_{wh1} < p_{who}$ and the bottom hole pressure dropped to $p_{bh1} < p_{bho}$.

Below are described the technique for the calculation of permeability and skin:

1. Record the initial stabilized wellhead pressure p_{who}.
2. Open the wellhead valve and produce blow off the gas with a very high flow rate for a very short time.
3. Close the wellhead valve and record the new wellhead pressure p_{wh1} (after closing the valve a pressure peak may be observed due to inertial forces which will decay after a while enabling recording of p_{wh1}).
4. Record the wellhead pressure build up versus time (p_{wh} vs. t) assuming p_{wh1} is p_{wh} for $t = 0$.
5. Calculate a

$$a = \frac{h\phi c \frac{m}{Z_{avg}RT_{avg}} \left(e^{\frac{mgH}{Z_{avg}RT_{avg}}}\right)^2 (p_{wh0} - p_{wh1})}{\left(e^{\frac{mgH}{Z_{avg}RT_{avg}}} - 1\right) \ln\left(\frac{p_{wh0}}{p_{wh1}}\right)} \tag{1}$$

6. Check if $a < \frac{1}{2}e$

If the above inequality doesn't hold the technique cannot be used.

7. Calculate $\overset{**}{u}$ using a simple iteration procedure where $\overset{**}{u}$ is the larger of two roots of the following equation:

$$u = \frac{1}{2}(\ln u - \ln a)$$

To find $\overset{**}{u}$ do the following:

- take for u any value from (1/2; ∞) range (for example $u_1 = 1$) and calculate $u_{i+1} = \frac{1}{2}(\ln u_i - \ln a$ where $i = 1, 2, \ldots, n$;
- if for i-th iteration $u_{i+1} - u_i < \varepsilon$ (where ε – assumed very small number) then $u_{i+1} = \overset{**}{u}$ (usually the few iterations are sufficient).

8. Mark the $\ln\left(\frac{p_{wh}(t) - p_{wh0}}{p_{wh1} - p_{wh0}}\right)$ vs. t data in rectangular system of coordinates and approximate the trajectory of points of a "long" time data using the straight line and the least squares method.

9. Record a slope E of a straight line and calculate the permeability using the equation (2) given below. Assume that the data which plot along the straight line for a "long" time of pressure buildup represent permeability of the reservoir [6, 7].

$$E = \frac{2khgm\left(e^{\frac{mgH}{Z_{avg}RT_{avg}}}\right)^2 (p_{wh0} - p_{wh1})}{\mu r_o^2 Z_{avg}RT_{avg}\left(e^{\frac{mgH}{Z_{avg}RT_{avg}}} - 1\right) \ln\left(\frac{p_{wh0}}{p_{wh1}}\right) \overset{**}{u}} \tag{2}$$

10. Calculate the skin factor S' by evaluating the point of intersection of a straight line for the "long" time data with $\ln\left(\frac{p_{wh}(t) - p_{wh0}}{p_{wh1} - p_{wh0}}\right)$ axis and $t = 0$, given as L, using the following equation:

$$S' = \overset{**}{u}(e^L - 1) \tag{3}$$

Example 1

Herein is an example for a gas well produced by Polish Oil and Gas Company. Blowing off some gas from the well which stabilized wellhead pressure was $p_{wh0} = 6.74$ MPa has caused the pressure to drop to 0.3 MPa at the moment of closing the well. The duration of flow preceding the pressure build up period was 90 seconds. Next the pressure immediately jumped to 4.18 MPa – which is attributed to inertial forces – and smoothly started to grow to initial wellhead value. The 4.18 MPa was assumed as initial pressure of build up period p_{wh1}. The recorded wellhead build up pressure versus time is shown in a table below. To simplify calculations of the molar mass we assumed that gas is composed of methane (80%) and ethane (20%). Such an assumption has rather minor impact on calculation results. The well is producing gas from sandstone rock. The remaining data are given below:

- Depth of the well $H = 3201$ m,
- Thickness of a gas zone $h = 15.0$ m,
- Initial wellhead pressure before some gas is blown off $p_{wh0} = 6.74$ MPa,
- Wellhead pressure after some gas was blown off $p_{wh1} = 4.18$ MPa,
- Molar mass $m = 18.445$ g/mol,
- Gas viscosity in reservoir conditions $\mu_g = 0.018$ cP,
- Total compressibility $c = 0.000902$ 1/MPa,
- Average reservoir temperature $T_{avg} = 386$ K,
- Average gas compressibility factor $Z_{avg} = 0.92$,
- Gas constant $R = 8314$ g m^2/(s^2 K mol),
- Porosity $\phi = 0.1532$,
- Well radius $r_o = 0.0745$ m.

Recorded relation of p_{wh} vs. t is shown in the Table 1 below and in Figure 1.

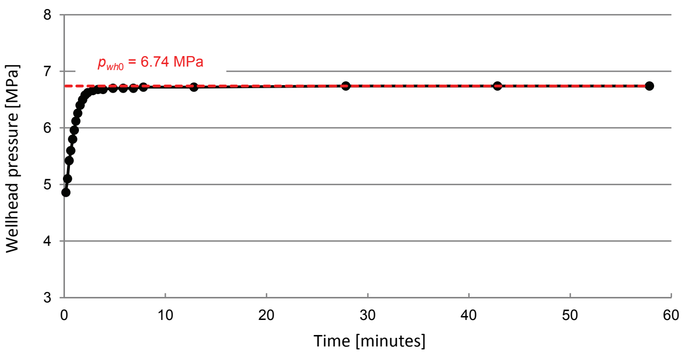

Fig. 1. Wellhead pressure versus time for example no. 1

Table 1. Recorded values p_{wh} vs. t

t [sec]	t [min]	$p_{wh}(t)$ [MPa]	$\ln p_D = \ln\left(\frac{p_{wh}(t) - p_{wh0}}{p_{wh1} - p_{wh0}}\right)$
10	0.17	4.86	–0.309
20	0.33	5.10	–0.445
30	0.50	5.42	–0.662
40	0.67	5.60	–0.809
50	0.83	5.80	–1.002
60	1.00	5.96	–1.188
70	1.17	6.12	–1.418
80	1.33	6.26	–1.674
95	1.58	6.40	–2.019
110	1.83	6.50	–2.367
125	2.08	6.58	–2.773
140	2.33	6.62	–3.060
170	2.83	6.66	–3.466
200	3.33	6.68	–3.753
230	3.83	6.68	–3.753
290	4.83	6.70	–4.159
350	5.83	6.70	–4.159
410	6.83	6.70	–4.159
470	7.83	6.72	–4.852
770	12.83	6.72	–4.852
1670	27.83	6.74	–
2570	42.83	6.74	–
3470	57.83	6.74	–

The procedure for calculation of permeability and skin is shown below:
Calculate „a" (Eq. 1):

$$a = 5.9 \cdot 10^{-4} \frac{h[\mathrm{m}]\phi c\left[\frac{1}{\mathrm{MPa}}\right] \frac{m\left[\frac{\mathrm{g}}{\mathrm{mol}}\right]}{Z_{avg}T_{avg}[\mathrm{K}]} \left(e^{\frac{1.18\cdot 10^{-3} m\left[\frac{\mathrm{g}}{\mathrm{mol}}\right] H[\mathrm{m}]}{Z_{avg}T_{avg}[\mathrm{K}]}}\right)^2 (p_{wh0} - p_{wh1})[\mathrm{MPa}]}{\left(e^{\frac{1.18\cdot 10^{-3} m\left[\frac{\mathrm{g}}{\mathrm{mol}}\right] H[\mathrm{m}]}{Z_{avg}T_{avg}[\mathrm{K}]}} - 1\right) \ln\left(\frac{p_{wh0}}{p_{wh1}}\right)}$$

and:

$$a = 2.325 \cdot 10^{-6} < \frac{1}{2e}$$

Calculate u using the iteration method provided in [6–8].
As the first approximation of u we assumed $u = 1$. After the four iterations we obtained for $\varepsilon = 0.01$:

$$\overset{**}{u} = 7.4929$$

Equation (2) converted to engineering units is given below.

$$E\left[\frac{1}{min}\right] = 1.42 \cdot 10^{-7} \frac{k[mD]h[\mathrm{m}]m\left[\frac{\mathrm{g}}{\mathrm{mol}}\right] \left(e^{\frac{1.18\cdot 10^{-3} m\left[\frac{\mathrm{g}}{\mathrm{mol}}\right] H[\mathrm{m}]}{Z_{avg}T_{avg}[\mathrm{K}]}}\right)^2 (p_{wh0} - p_{wh1})[\mathrm{MPa}]}{\mu[\mathrm{cP}] r_o^2[\mathrm{m}^2] Z_{avg}T_{avg}[\mathrm{K}] \left(e^{\frac{1.18\cdot 10^{-3} m\left[\frac{\mathrm{g}}{\mathrm{mol}}\right] H[\mathrm{m}]}{Z_{avg}T_{avg}[\mathrm{K}]}} - 1\right) \ln\left(\frac{p_{wh0}}{p_{wh1}}\right) \overset{**}{u}}$$

Mark the $\ln\left(\frac{p_{wh}(t)-p_{wh0}}{p_{wh1}-p_{wh0}}\right)$ versus t (Figure 2) in rectangular system of coordinates and approximate trajectory of data using the straight line and the least squares method. Record the slope E of this line and calculate permeability using Eq. 2. As shown in Figure 1 the pressure growth is initially rapid and slows down after approximately 200 seconds. Afterwards, the pressure increase is very slow (0.06 MPa during 55 minutes). The slope of the straight line of $\ln\left(\frac{p_{wh}(t)-p_{wh0}}{p_{wh1}-p_{wh0}}\right)$ vs. t relation is $E = -0.1117$ 1/min and point of intersection of this line with $\ln\left(\frac{p_{wh}(t)-p_{wh0}}{p_{wh1}-p_{wh0}}\right)$ axis for $t = 0$, given as L, is $L = -3.44$ which gives the permeability of the reservoir $k = 20.65$ mD and $S' = -7.25$.

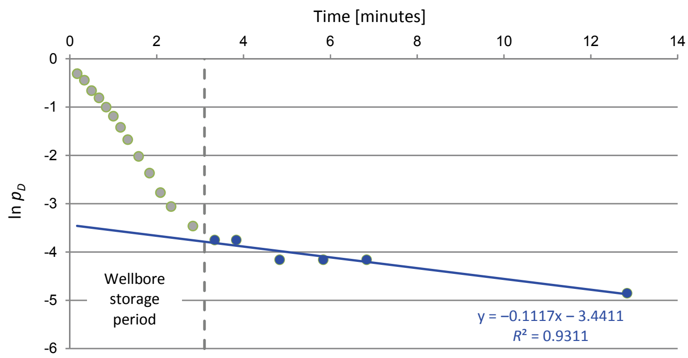

Fig. 2. ln p_D versus time

Example 2

Here is another gas well produced by Polish Oil and Gas Company. The reservoir rock was the sandstone. As before, quick blowing off of some gas from the well which stabilized wellhead pressure was $p_{wh0} = 24.1$ MPa caused the pressure to drop to 1.182 MPa. The recorded wellhead pressure versus time is shown in table 2 below. Because no gas composition was given we assumed that the gas molar mass is $m = 18.445$ g/mol. The remaining data is as follows:

- Well depth $H = 3310$ m,
- Gas zone thickness $h = 12.0$ m,
- Initial shut in wellhead pressure (before the gas is blown off the well) $p_{wh0} = 24.1$ MPa,
- Wellhead pressure after gas is blown off and the well is closed $p_{wh1} = 1.182$ MPa,
- Molar mass $m = 18.445$ g/mol,
- Gas viscosity in reservoir conditions $\mu_g = 0.018$ cP,
- Total compressibility factor $c = 0.003$ 1/MPa,
- Average reservoir temperature $T_{avg} = 373$ K,
- Gas deviation factor $Z_{avg} = 0.92$,
- Porosity $\phi = 0.15$,
- Well radius $r_o = 0.057$ m.

The recorded wellhead pressure build up versus time is shown in the table below and in Figure 3.

Table 2. Recorded values p_{wh} vs. t

t [min]	$p_{wh}(t)$ [MPa]	$\ln p_D = \ln\left(\frac{p_{wh}(t)-p_{wh0}}{p_{wh1}-p_{wh0}}\right)$
2	3.97	–0.1298
3	8.48	–0.3833
4	12.18	–0.6540
5	14.83	–0.9048
6	16.29	–1.0764
7	17.09	–1.1846
8	17.99	–1.3223
9	18.43	–1.3959
10	18.94	–1.4909
12	19.43	–1.5901
14	19.96	–1.7115
16	20.33	–1.8043
18	20.70	–1.9067
20	21.08	–2.0273
22	21.16	–2.0544
24	21.36	–2.1250
26	21.50	–2.1749
28	21.70	–2.2551
30	21.94	–2.3918
40	22.57	–2.7073
50	22.96	–2.9994
60	23.23	–3.2736
70	23.46	–3.5767
80	23.63	–3.8925
90	23.77	–4.2491

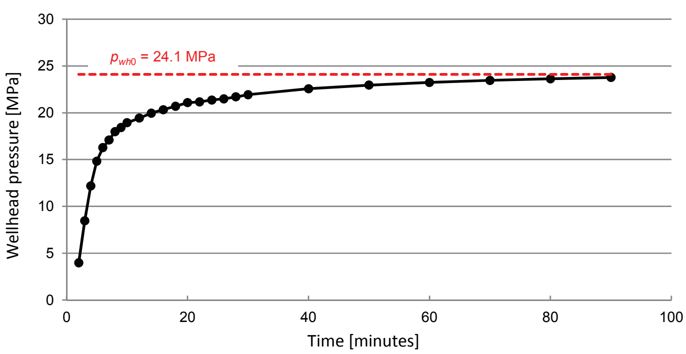

Fig. 3. Wellhead pressure build up versus time

Calculate a using Eq. (1):

$$a = 8.479 \cdot 10^{-6} < \frac{1}{2e}$$

As the first approximation of u we assumed $u = 1$. After five iterations we have for $\varepsilon = 0.01$:

$$\overset{**}{u} = 6.7972$$

Mark the $\ln\left(\frac{p_{wh}(t) - p_{wh0}}{p_{wh1} - p_{wh0}}\right)$ versus t in the rectangular system of coordinates and approximate trajectory of data using the straight line and the least squares method. Record the slope E of this line and calculate permeability k using Eq. (2).The slope of the straight line is $E = -0.0317$ 1/min, and permeability $k = 2.78$ mD.

The point of intersection of the straight line with the $\ln\left(\frac{p_{wh}(t) - p_{wh0}}{p_{wh1} - p_{wh0}}\right)$ axis and $t = 0$ is $L = -1.3785$.

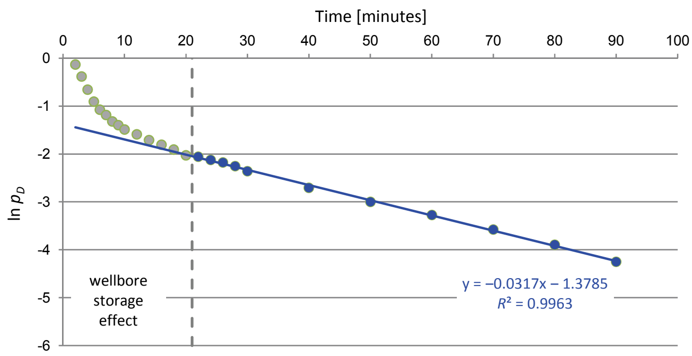

Fig. 4. ln p_D versus time

Calculate the skin factor using the following formula: $S' = \overset{**}{u}\,(e^L - 1) = 6.7972\,(e^{-1.3785} - 1) = -5.08$

Example 3

Herein is another example of POGC well. The two flow and two build up tests were carried out in a gas well completed in very low permeability limestone. The wellhead build up pressure vs. time was recorded for the two buildup periods. Each buildup period was preceded by the very short gas flow period. Below you will find the data for each pressure buildup period plus calculation of the permeability and skin factor for the reservoir. The permeability of the reservoir evaluated later using the Horner method, is 0.0229 mD and 0.0224 mD for the first and second build up period respectively. The data is as follows:

- Well depth $H = 3741$ m,
- Gas zone thickness $h = 82.0$ m,
- Initial shut in wellhead pressure (before the gas is blown off the well) $p_{wh0} = 32.9$ MPa,
- Wellhead pressure after gas is blown off and the well is closed $p_{wh1} = 14.8$ MPa,
- Molar mass $m = 18.445$ g/mol,
- Gas viscosity in reservoir conditions $\mu_g = 0.018$ cP,
- Total compressibility factor $c = 0.003$ 1/MPa,
- Average reservoir temperature $T_{avg} = 374$ K,
- Gas deviation factor $Z_{avg} = 0.97$,
- Porosity $\phi = 0.05$,
- Well radius $r_o = 0.108$ m.

Table 3. Recorded values p_{wh} vs. t for the first build up period

t [min]	$p_{wh}(t)$ [MPa]	$\ln p_D = \ln\left(\frac{p_{wh}(t) - p_{wh0}}{p_{wh1} - p_{wh0}}\right)$
7	15.01	–0.012
12	15.30	–0.028
17	15.49	–0.039
27	16.00	–0.069
37	16.35	–0.090
52	17.05	–0.133
72	18.01	–0.195
92	18.96	–0.261
122	20.44	–0.373
152	21.90	–0.498
186	23.35	–0.639
227	25.31	–0.869
257	27.26	–1.166
352	31.00	–2.253
467	32.27	–3.357
527	32.31	–3.422
647	32.41	–3.601
722	32.45	–3.703

First build up period

The relation between the wellhead build up pressure and time is shown in Table 3 and in Figure 5.

Calculate a using Eq. (1):

$$a = 5.205 \cdot 10^{-5} < \frac{1}{2e}$$

As first approximation of u we assumed $u = 1$. After five iterations we have for $\varepsilon = 0.01$:

$$\overset{**}{u} = 5.8116$$

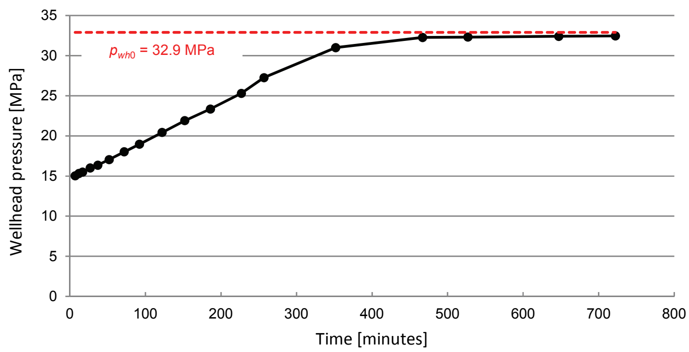

Fig. 5. Wellhead pressure build up versus time

Mark the $\ln\left(\frac{p_{wh}(t)-p_{wh0}}{p_{wh1}-p_{wh0}}\right)$ versus t in rectangular system of coordinates and approximate trajectory of data using the straight line and least squares method. Record the slope E of this line and calculate permeability k using Eq. (2). The slope of the straight line is E = –0.0014 1/min, and permeability k = 0.02 mD.

The point of intersection of this line with the $\ln\left(\frac{p_{wh}(t)-p_{wh0}}{p_{wh1}-p_{wh0}}\right)$ axis for the "long" time data and t = 0 is L = –2.7029.

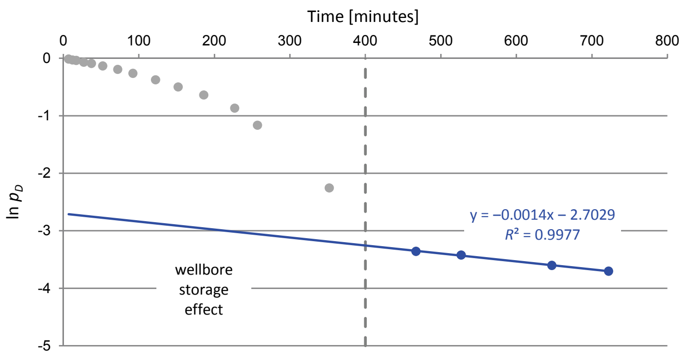

Fig. 6. ln p_D versus time

Calculate the skin factor using the following formula:

$$S' = \overset{**}{u}\,(e^L - 1) = 5.8116\,(e^{-2.7029} - 1) = -5.42$$

Second build up period

The relation between the wellhead build up pressure and time is shown in Table 4 and in Figure 7.

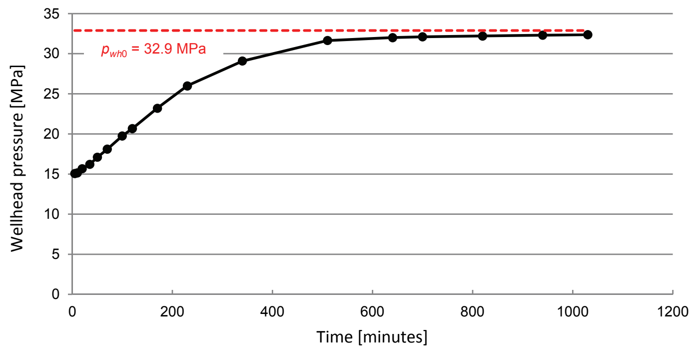

Fig. 7. Wellhead pressure build up versus time

Table 4. Recorded values p_{wh} vs. t for the first build up period

t [min]	$p_{wh}(t)$ [MPa]	$\ln p_D = \ln\left(\frac{p_{wh}(t)-p_{wh0}}{p_{wh1}-p_{wh0}}\right)$
5	15.050	–0.014
10	15.150	–0.020
20	15.674	–0.049
35	16.205	–0.081
50	17.095	–0.136
70	18.105	–0.202
100	19.755	–0.320
120	20.656	–0.391
170	23.206	–0.624
230	25.997	–0.964
340	29.090	–1.558
510	31.660	–2.681
640	32.009	–3.011
700	32.105	–3.125
820	32.209	–3.265
940	32.309	–3.422
1030	32.369	–3.529

Calculate a using Eq. (1):

$$a = 5.205 \cdot 10^{-5} < \frac{1}{2e}$$

As first approximation of u we assumed u = 1. After five iterations we have for ε = 0.01:

$$\overset{**}{u} = 5.8116$$

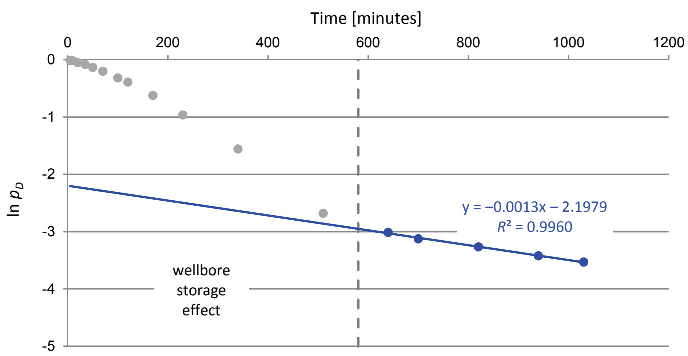

Fig. 8. ln p_D versus time

Mark the $\ln\left(\frac{p_{wh}(t)-p_{wh0}}{p_{wh1}-p_{wh0}}\right)$ versus t in the rectangular system of coordinates and approximate trajectory of data using the straight line and the least squares method. Record the slope E of this line and calculate the permeability k using Eq. (2). The slope of the straight line for the "long" time data which represent reservoir is E = –0.0013 1/min, and permeability k = 0.019 mD. The point of intersection of the this straight line with the $\ln\left(\frac{p_{wh}(t)-p_{wh0}}{p_{wh1}-p_{wh0}}\right)$ axis for the "long" time data and t = 0 is L = –2.1979.

Calculate the skin factor using the following formula:

$$S' = \overset{**}{u}\,(e^{L} - 1) = 5.8116\,(e^{-2.1979} - 1) = -5.17$$

As shown, in the case being analyzed, the permeability calculated using the Horner method and using the presented technique are in agreement.

Example 4

Here is a next example of a gas well drilled by POGC. The well data are given below:

- Well depth $H = 2100$ m,
- Gas zone thickness $h = 50.0$ m
- Initial shut in wellhead pressure (before the gas is blown off the well) $p_{wh0} = 16.102$ MPa
- Wellhead pressure after gas is blown off and the well is closed $p_{wh1} = 15.583$ MPa
- Molar mass $m = 16.223$ g/mol
- Gas viscosity in reservoir conditions $\mu_g = 0.0174$ cP
- Total compressibility factor $c = 0.00102$ 1/MPa
- Average reservoir temperature $T_{avg} = 347$ K
- Gas deviation factor $Z_{avg} = 0.97$
- Porosity $\phi = 0.045$
- Well radius $r_o = 0.112$ m

The relation of wellhead pressure build up versus time is shown in Figure 9 and relation the $\ln_{p_D}$ versus t is shown in Figure 10.

Calculate a using Eq. (1):

$$a = 1.035 \cdot 10^{-5} < \frac{1}{2e}$$

As the first approximation of u we assumed $u = 1$. After five iterations we have for $\varepsilon = 0.01$:

$$\overset{**}{u} = 6.6896$$

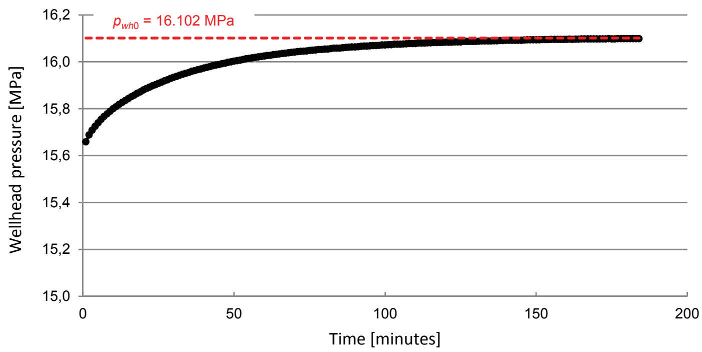

Fig. 9. Wellhead pressure build up versus time

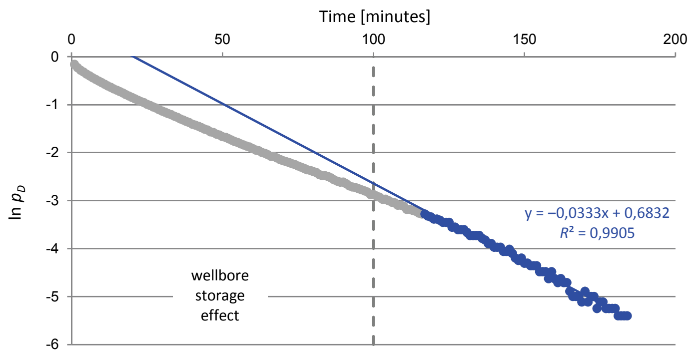

Fig. 10. ln p_D versus time

The slope of the straight line $\ln\left(\frac{p_{wh}(t) - p_{wh0}}{p_{wh1} - p_{wh0}}\right)$ versus t is $E = -0.0333$ 1/min. The calculated permeability of a pay zone is $k = 0.90$ mD and S' = 6.56.

Example 5

Here is a next example of a POGC gas well. The well data are given below:

- Well depth $H = 2245$ m,
- Gas zone thickness $h = 92.0$ m,
- Initial shut in wellhead pressure (before the gas is blown off the well) $p_{wh0} = 16.046$ MPa,
- Wellhead pressure after gas is blown off and the well is closed $p_{wh1} = 12.766$ MPa,
- Molar mass $m = 16.252$ g/mol,
- Gas viscosity in reservoir conditions $\mu_g = 0.0174$ cP,
- Total compressibility factor $c = 0.000839$ 1/MPa,
- Average reservoir temperature $T_{avg} = 344$ K,
- Gas deviation factor $Z_{avg} = 0.97$,

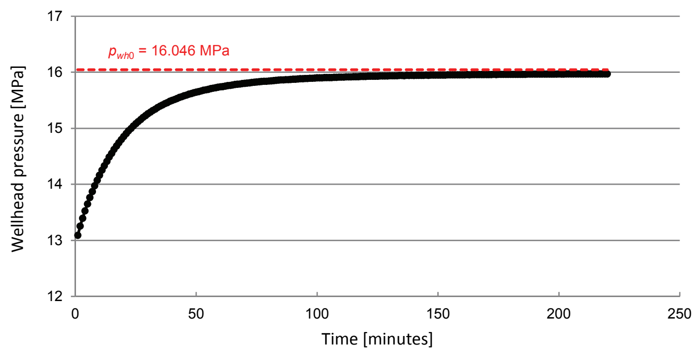

Fig. 11. Wellhead pressure build up versus time

- Porosity $\phi = 0.068$,
- Well radius $r_o = 0.079$ m.

The relation of wellhead pressure build up versus time is shown in Figure 11 and relation the ln p_D versus t is shown in Figure 12.

Calculate a using Eq. (1) is $a = 2.033 \cdot 10^{-5} < \frac{1}{2e}$. As the first approximation of u we assumed $u = 1$. After five iterations we have for $\varepsilon = 0.01$:

$$\overset{**}{u} = 6.3238$$

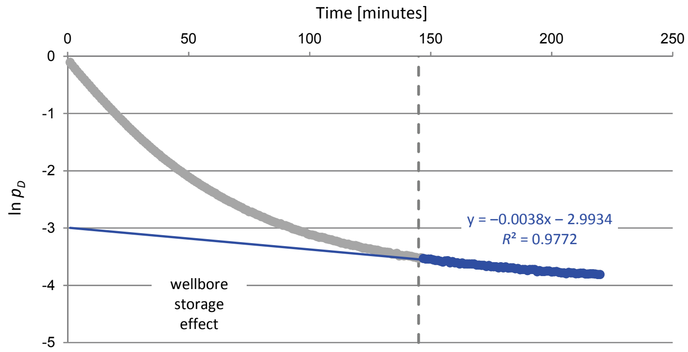

Fig. 12. ln p_D versus time

The slope of the straight line ln $\left(\frac{p_{wh}(t) - p_{wh0}}{p_{wh1} - p_{wh0}}\right)$ versus t is $E = -0.0038$ 1/min. The calculated permeability of a pay zone is $k = 0.031$ mD and S' = –6.01.

Advantages of presented technique

1. Method is relatively simple.
2. Practically costless.
3. Neither gas flow rate measurement nor installation of downhole gauge is needed.
4. Easy to use (only wellhead pressure measurements are required).
5. Interpretation of data is easy.

Disadvantages

1. Results are approximate.
2. The presented procedure has not been verified extensively.

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Appendix A

Derivation of Equation relating the wellhead pressure build up versus time

Consider a dry gas well in which the wellhead pressure p_{who} plus pressure exerted by compressed gas is equal to the reservoir pressure $p_{bho} = p_o$, i.e. stabilized pressure conditions exists. If the wellhead valve is open and gas is produced blown off for a while and the well is closed afterwards, the initial wellhead pressure p_{who} becomes $p_{wh1} < p_{who}$ and the initial bottomhole pressure p_{bho} becomes $p_{bh1} < p_{bho} = p_o$. Theoretically the pressures within the well will stabilize again at their original values after infinite time, i.e. p_{wh1} would build up to p_{who} and p_{bh1} would build up to $p_{bho} = p_o$ – the reservoir pressure.

We use the mathematical model given in [8] (which has been modified by us) to describe the situation depicted above and to account for the presence of gas in the well.

- Below we recall the major assumptions of this moThe rate of pressure change within gas reservoir is in direct proportion to the difference between actual pressure $p(r, t)$ and initial reservoir pressure $p_o = p_{bho}$.

$$\frac{\partial p}{\partial t} = -E\left(p(r,t) - p_o\right) \tag{A.1}$$

which means that:

$$p(r,t) = p_o + Ce^{-Et} \tag{A.2}$$

where C does not depend on t.

- The pressure within the reservoir satisfies the diffusivity equation:

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{\phi \mu C}{k}\frac{\partial p}{\partial t} \tag{A.3}$$

and so $C(r)$ satisfies the following equation:

$$C''(r) + \frac{1}{2}C'(r) + \frac{\phi \mu c}{k}EC(r) = 0 \tag{A.4}$$

The solution of (A.4) is:

$$C(r) = a_1 J_o\left(r\sqrt{\frac{\phi \mu c E}{k}}\right) + a_2 Y_o\left(r\sqrt{\frac{\phi \mu c E}{k}}\right) \tag{A.5}$$

where J_o and Y_o are Bessel functions of the first kind, zero order [2].

- Assume $a_1 = 0$ in equation (A.5) because for a small r the quantity $r\sqrt{\frac{\phi \mu c E}{k}}$ is very small in all practical applications and for small arguments the Y_o function is much greater than J_o ($J_o(0) = 1$). Substitution the Eq. (A.5) into Eq. (A.2) yields for $a_1 = 0$

$$p(r,t) = p_o + a_2 Y_o\left(r\sqrt{\frac{\phi \mu c E}{k}}\right)e^{-Et} \tag{A.6}$$

where p_0 is a initial reservoir pressure equal to p_{bho} and $p(r,t)$ is pressure within reservoir.

- Assume that $p_{bh}(r_o, 0)$ (i.e. the pressure after some gas was blown off the well and the wellhead valve was closed) is equal to p_{bh1} and so a_2 is as given below:

$$a_2 = \frac{p_{bh1} - p_{bh0}}{Y_o\left(r_o\sqrt{\frac{\phi \mu c E}{k}}\right)} \tag{A.7}$$

Substitution of (A.7) into (A.8) gives:

$$p(r,t) = p_{bho} + \frac{p_{bh1} - p_{bho}}{Y_o\left(r_o\sqrt{\frac{\phi \mu c E}{k}}\right)} Y_o\left(r\sqrt{\frac{\phi \mu c E}{k}}\right)e^{-Et} \tag{A.8}$$

- The following approximation holds for small values of argument of Y_o [2, 5].

$$Y_o\left(r_o\sqrt{\frac{\phi\mu cE}{k}}\right)=-\frac{2}{\pi}\ln\left(\frac{r_o}{2}\sqrt{\frac{\phi\mu cE}{k}}\right) \tag{A.9}$$

and so finally we have:

$$p(r,t)=p_{bh0}+\frac{p_{bh1}-p_{bh0}}{\ln\left(\frac{r_o}{2}\sqrt{\frac{\phi\mu cE}{k}}\right)}\ln\left(\frac{r}{2}\sqrt{\frac{\phi\mu cE}{k}}\right)e^{-Et} \tag{A.10}$$

The velocity of gas flow at the borehole wall is given by:

$$v(r_0,t)=-\frac{k}{\mu}\frac{p_{bh1}-p_{bh0}}{\ln\frac{r_0}{2}\sqrt{\frac{\phi\mu cE}{k}}}\frac{1}{r}e^{-Et} \tag{A.11}$$

The density of gas within the well is given by the following equation:

$$\rho_g(z,t)=\frac{mp_g(z,t)}{Z_{avg}RT_{avg}} \tag{A.12}$$

- Assume that the gas flow friction is negligible and that the gas pressure within the static gas column at any depth z is given by [1]:

$$p_g(z,t)=p_{wh}(t)e^{\frac{mgz}{Z_{avg}RT_{avg}}} \tag{A.13}$$

where $p_{wh}(t)$ – gas wellhead pressure
$p_g(z,t)$ – gas pressure at depth z

The mass of the gas within the well at any time t is equal to:

$$M_g(t)=\int_0^H\rho_g(z,t)\pi r_0^2dz \tag{A.14}$$

Combining the Equations (A.12), (A.13) and (A.14), the rate of mass growth within the well is:

$$\frac{dM_g}{dt}=\frac{\pi r_0^2}{g}\left(e^{\frac{mgH}{Z_{avg}RT_{avg}}}-1\right)\left(\frac{dp_{wh}(t)}{dt}\right) \tag{A.15}$$

On the other hand the rate of the gas mass flow from the reservoir is equal to:

$$\frac{dM_g}{dt}=2\pi r_0hv(r_0,t)\rho_g(H,t) \tag{A.16}$$

Substitution of (A.12), (A.13) and (A.11) into (A.16) yields:

$$\frac{dM_g}{dt}=2\pi h\frac{k}{\mu}\frac{m}{Z_{avg}RT_{avg}}\frac{\left(e^{\frac{mgh}{Z_{avg}RT_{avg}}}\right)^2(p_{wh1}-p_{wh0})p_{wh}(t)}{\ln\left(\frac{r_0}{2}\sqrt{\frac{\phi\mu cE}{k}}\right)}e^{-Et} \tag{A.17}$$

Comparing (A.17) and (A.15) gives the following relations:

$$\frac{dp_{wh}(t)}{p_{wh}(t)}=-A\frac{e^{-Et}dt}{\ln\left(\frac{r_0}{2}\sqrt{\frac{\phi\mu cE}{k}}\right)} \tag{A.18}$$

where

$$A = \frac{2hk}{\mu r_0^2} \frac{mg}{Z_{avg} RT_{avg}} \frac{\left(e^{\frac{mgH}{Z_{avg}RT_{avg}}}\right)^2 (p_{wh1} - p_{wh0})}{\left(e^{\frac{mgH}{Z_{avg}RT_{avg}}} - 1\right)} \tag{A.19}$$

In Equations (A.15) to (A.19) the bottom hole pressures have been converted to the wellhead pressures using Equation (A.13). Because we assumed that:

$$\text{for } t = 0 \qquad p_{wh} = p_{wh1}$$
$$\text{for } t \to \infty \qquad p_{wh} = p_{wh0}$$

so we can write:

$$\int_{p_{wh1}}^{p_{wh0}} \frac{dp_{wh}(t)}{p_{wh}(t)} = -A \frac{1}{\ln\left(\frac{r_0}{2}\sqrt{\frac{\phi\mu cE}{k}}\right)} \int_0^{\infty} e^{-Et} dt \tag{A.20}$$

Solving equation (A.20) we got after rearrangement of terms:

$$\sqrt{\frac{r_0^2 \phi\mu cE}{4k}} = e^{\frac{-A}{E \ln\left(\frac{p_{wh0}}{p_{wh1}}\right)}} \tag{A.21}$$

Equation (A.21) can be presented in the following form:

$$e^{-u} = \sqrt{\frac{a}{u}} \tag{A.22}$$

or

$$ue^{-2u} = a$$

where a is given below:

$$a = \frac{h\phi c \frac{mg}{Z_{avg}RT_{avg}} \left(e^{\frac{mgH}{Z_{avg}RT_{avg}}}\right)^2 (p_{wh0} - p_{wh1})}{2\left(e^{\frac{mgH}{Z_{avg}RT_{avg}}} - 1\right) \ln\left(\frac{p_{wh0}}{p_{wh1}}\right)} \tag{A.23}$$

and

$$u = -\ln\sqrt{\frac{r_0^2 \phi\mu cE}{4k}} \tag{A.24}$$

Equation (A.22) has its extreme value for $u = 1/2$ because

$$\left(\frac{dy}{du}\right)_{u=1/2} = e^{-2u}(1-2u) = 0 \tag{A.25}$$

and

$$y\left(u = \frac{1}{2}\right) = \frac{1}{2e} \tag{A.26}$$

The visualization of $y(u)$ is shown in Fig. A.1

It is evident from Fig. A.1 that:
- for $a > 1/2e$ Equation (A.25) has no roots,
- for $a = 1/2e$ Equation (A.25) has one root $u = 1/2$,
- for $a < 1/2e$ Equation (A.25) has two roots,
- first root of Equation (A.25) lies within (0, 1/2) interval.

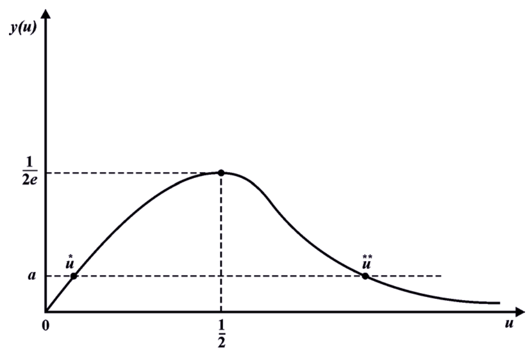


Fig. A.1. Visualization of $y(u)$ function

The value of a is very small in the practical applications and so Equation (A.25) has two roots. The first root $\overset{*}{u} \in (0,1/2)$ should be discarded because the value of $\frac{r_o}{2}\sqrt{\frac{\phi\mu c E}{k}}$ would be too large to justify the approximation shown in Equation (A.9). To calculate the second root $\overset{**}{u}$ we present Equation (A.22) in the following form:

$$u = \frac{1}{2}\ln u - \frac{1}{2}\ln a \tag{A.27}$$

and use the simple iteration procedure shown below:

1. Let any value of u from $\left(\frac{1}{2};\infty\right)$ interval be a first approximation of $\overset{**}{u}$. We have from the Equation (A.27)

$$u_{i+1} = \frac{1}{2}\ln u_i - \frac{1}{2}\ln a \tag{A.28}$$

   where $i = 1, 2, \ldots, n$.

2. If $u_{i+1} - u_i \le \varepsilon$, where ε is arbitrary selected small value, then the iteration is terminated and $u_{i+1} = \overset{**}{u}$. Usually a few iterations are sufficient to calculate $\overset{**}{u}$.

Knowing $\overset{**}{u}$ and combining Equations (A.21), (A.24) and (A.19) we can provide the Equation for E:

$$E = \frac{k}{\mu}\frac{2hgm}{r_0^2 Z_{avg} RT_{avg}} \frac{\left(e^{\frac{mgH}{Z_{avg}RT_{avg}}}\right)^2 (p_{wh0} - p_{wh1})}{\left(e^{\frac{mgH}{Z_{avg}RT_{avg}}} - 1\right)\ln\left(\frac{p_{wh0}}{p_{wh1}}\right)} \frac{1}{\overset{**}{u}} \tag{A.29}$$

The permeability around the wellbore is usually different than those of the reservoir due to mud and rock interaction or because of mechanical reasons. The difference between the measured and the theoretical gas buildup pressure is caused by the so called skin effect, which accounts for the permeability of the wellbore zone. The divergence between the measured gas buildup pressure and the theoretical value, is attributed to so called skin effect, which is defined using the following formula:

$$\Delta p_{skin} = \frac{Q\mu S}{2\pi kh} \tag{A.30}$$

The gas flow rate Q may be expressed using (A.11) as:

$$Q = 2\pi r_0 h v(r_0, t) = 2\pi r_0 h\left(\frac{-k}{\mu}\right)\frac{p_{bh1} - p_{bh0}}{\ln\left(\frac{r_0}{2}\sqrt{\frac{\phi\mu c t}{k}}\right)}\frac{1}{r_0}e^{-Et} \tag{A.31}$$

Combining the equations (A.31) and (A.21) and including Δp_{skin} in equation (A.8) we get the following relation:

$$p(r,t) = p_{bh0} + \frac{p_{bh1} - p_{bh0}}{\ln\left(\frac{r_0}{2}\sqrt{\frac{\phi\mu c E}{k}}\right)}\ln\left(\frac{r}{2}\sqrt{\frac{\phi\mu c t}{k}}\right)e^{-Et} - \frac{p_{bh1} - p_{bh0}}{\ln\left(\frac{r_0}{2}\sqrt{\frac{\phi\mu c E}{k}}\right)}e^{-Et}S \tag{A.32}$$

Converting the bottom hole pressures to the wellhead pressures using Equation (A.13) we got for $r = r_o$ accounting for the Equation (A.24)

$$\frac{p_{wh}(t) - p_{wh0}}{p_{wh1} - p_{wh0}} = e^{-Et}\left[1 + \frac{S}{\overset{**}{u}}\right] \tag{A.33}$$

or:

$$\ln\left(\frac{p_{wh}(t)-p_{wh0}}{p_{wh1}-p_{wh0}}\right)=-Et+\ln\left(1+\frac{S}{\overset{**}{u}}\right) \tag{A.34}$$

The Equation (A.34) may be also presented in a following form:

$$\ln p_D = -\frac{4a}{\overset{**}{u}}\ln t_D + \ln\left(1+\frac{S}{\overset{**}{u}}\right) \tag{A.35}$$

where

$p_D = \dfrac{p_{wh}(t)-p_{wh_0}}{p_{wh_1}-p_{wh_0}}$ – dimensionless pressure

$t_D = \dfrac{kt}{\phi\mu c r_0^2}$ – dimensionless time

The Equation (A.34) indicates that $p_D = \dfrac{p_{wh}(t)-p_{wh_0}}{p_{wh_1}-p_{wh_0}}$ vs. t data should plot along the straight line with the slope E enabling calculation of the permeability using Equation (A.30) and that the point of the intersection of this line with the $p_D = \dfrac{p_{wh}(t)-p_{wh_0}}{p_{wh_1}-p_{wh_0}}$ axis at $t = 0$ – given as L – enables calculation of a skin effect using the equation given below:

$$S' = \overset{**}{u}(e^L - 1) \tag{A.36}$$

To improve the accuracy of the permeability calculation we used the trial and error method described in [1] for evaluation of Z_{avg} of each pressure record. The final Z_{avg} and T_{avg} for the whole test are calculated as (for example) the arithmetic mean of all $Z_{avg\,i}$ and $T_{avg\,i}$.

In Equation (A.31) the S' is a rate dependent skin given by: $S' = S + DQ$. Because the flow rate of gas (within the closed well) during pressure build up period is a time dependent function, so it is not possible to calculate the components of S using data of a single build up test. In the test being discussed the S' reflects the impact which the time dependent flow rate Q and the mechanical skin S have on the build up pressure behavior.

To evaluate S and D, the two build up tests should be run using two different initial wellhead pressures p_{wh1} and p_{wh2}. Next the S_1 and S_2 should be calculated taking Q_1 and Q_2 (Eq. (A.32)) for $t = 0$ and p_{wh1} and p_{wh2} for the first and second build up period respectively.

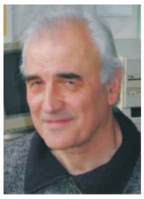
In the mathematical model we assumed that the speed of pressure change within reservoir is in direct proportion to the difference between the actual reservoir pressure and the initial reservoir pressure which means that the wellbore zone will react first if the wellhead valve is closed. The speed of the pressure change is the greatest around the wellbore and so the early time build up pressure behavior is dominated by the properties of this zone plus inertial forces, pressure fluctuations and the changes of gas temperatures and changes of gas deviation factor within the well, which (in the majority of cases) makes the data of this period unsuitable for interpretation. At the beginning of the pressure build up period, the gas accumulation within the well has dominant influence on the wellhead pressure behavior. This influence decays in time and, later on, the reservoir should start to behave according to the model "as a whole" enabling the calculation of its permeability. This initial period is called the "wellbore storage period".

The partial evacuation of gas from the well should be done "as quickly as possible" i.e. during a dozen or so seconds because of the following reason:

The E value (Eq. A.30) is very small in all practical application (specifically for small kh (mD m)) and so for the very short flow period preceding the well closure the exp(-Et) is very close to unity as it is for $t = 0$. The conclusion is that we can simulate the pressure buildup using Eq.A.10 assuming the wellhead valve is closed at $t = 0$. The same approach is adopted in the well known "slug test" method which is used for calculation of permeability and skin of horizons which do not flow to the surface.

Nomenclature

p_{who} – initial stabilized wellhead pressure
p_{bho} – initial stabilized downhole pressure equal to reservoir pressure
p_{wh1} – wellhead pressure after some gas was blown from the well and wellhead valve was closed
$p_{wh}(t)$ – actual wellhead pressure (p_{wh} $(t = 0)) = p_{wh1}$
z – depth
H – depth of a well
h – thickness of gas layer
μ_g – gas viscosity at reservoir conditions
c – total compressibility
Z_{avg} – average value of Z factor
T_{avg} – average temperature within the well
R – gas constant
ϕ – porosity of gas reservoir
r_o – well radius
r – radius
$p_g(z,t)$ – gas pressure within the well
$\rho_g(z,t)$ – gas density within the well



Dr inż. Tadeusz SZPUNAR
Adiunkt w Zakładzie Inżynierii Naftowej
Instytut Nafty i Gazu – Państwowy Instytut Badawczy
ul. Lubicz 25 A
31-503 Kraków
E-mail: *tadeusz.szpunar@inig.pl*

Mgr inż. Paweł BUDAK
Starszy specjalista naukowo-badawczy, kierownik Zakładu Inżynierii Naftowej
Instytut Nafty i Gazu – Państwowy Instytut Badawczy
ul. Lubicz 25 A
31-503 Kraków
E-mail: *pawel.budak@inig.pl*

Mgr inż. Mariusz DZIADKIEWICZ
Dyrektor Eksploatacji Złóż
Polskie Górnictwo Naftowe i Gazownictwo S.A.
Oddział Geologii i Eksploatacji
ul. Marcina Kasprzaka 25A
01-224 Warszawa